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The optimum arrangement of the plates in a multi-layered shield

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Abstract

A normal impact of a three-dimensional rigid conical impactor penetrating into a layered shield is studied using a simplified model for an impactor–shield interaction. The shield consists of adjacent plates manufactured from one of two possible materials, and the total thickness of the plates manufactured from every material is given. It is found that advancing any plate inside a shield in the direction of penetration causes a monotone change in the ballistic limit velocity. A criterion for increasing or decreasing of the ballistic limit velocity, which depends on the properties of the materials of the layers in the shield, is determined. A maximum ballistic limit velocity is attained for a two-layered shield without alternating the plates manufactured from different materials. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Analysis of research on sub-ordnance penetration and perforation of non-homogeneous (layered) shields can be found in the review by Corbett et al. (1996) and in the monograph by Arbate (1998) which can be supplemented by several recent publications (Almohandes et al., 1996; Ben-Dor et al., 1997a, 1998a; Gupta and Madhu, 1997; Mileiko et al., 1994; Nixdorff, 1987; Weidemaier et al., 1993). A number of studies are concerned directly with the optimization of multi-layered shields (e.g., Aptukov et al., 1992; Hetherington, 1992; Shupikov et al., 1996; Wang and Lu, 1996).

A conclusion can be made that only using simplified models allows us to analytically determine the

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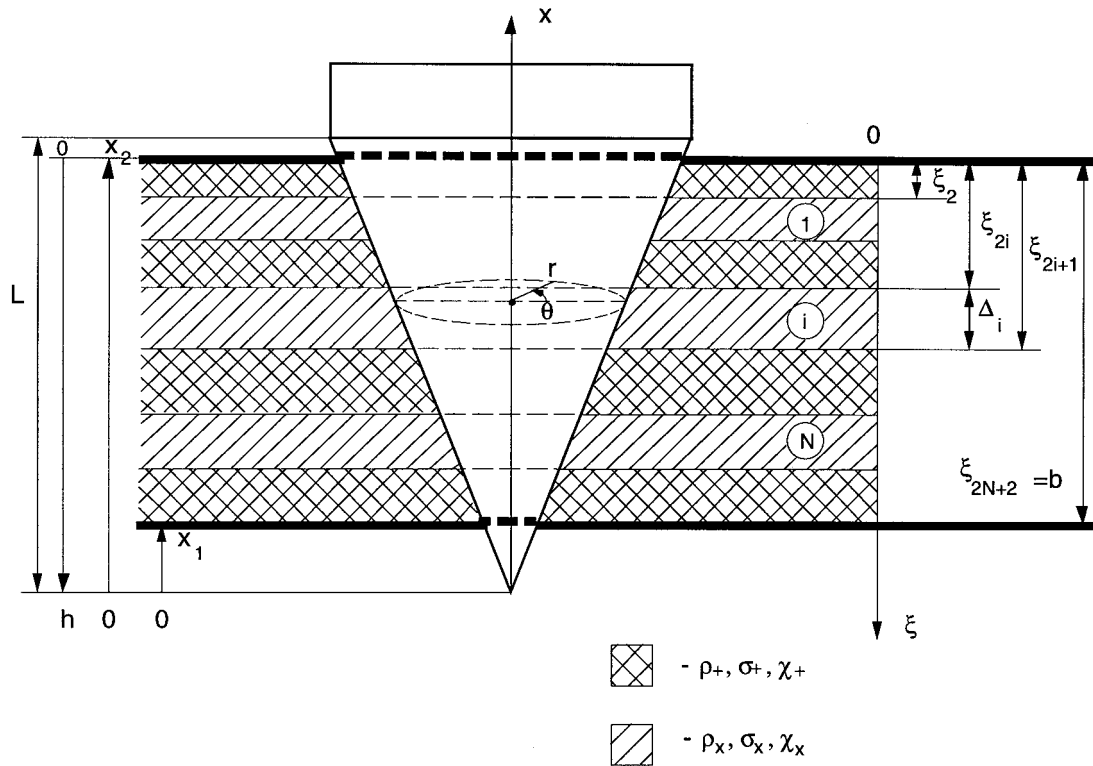


Fig. 1. Coordinates and notations.

laws which can be applied for further theoretical and experimental investigations. The localized impactor–shield interaction approach (Bunimovich and Dubinsky, 1995; Recht, 1990) was demonstrated to have some advantages in this respect when applied to the problems of penetration mechanics (Ben-Dor et al., 1997a, 1997b, 1998a, 1998b, 1998c; Ostapenko et al., 1994; Ostapenko and Yakunina, 1997; Vedernikov and Shchepanovsky, 1995). Aptukov et al. (1992) used localized interaction models in determining minimum weight inhomogeneous shields using two basic approaches, continuous and discrete. In the first approach, it was assumed that density is the only parameter which determines the mechanical properties of the material of a shield, and Pontryagin's Maximum Principle was used to determine the optimum distribution of density in the shield. In the second approach, a shield was considered to be assembled from layers manufactured from a given set of materials which were characterized by several independent parameters, and special variational techniques were used. When an impactor reached the rear surface of a shield, a shield was considered perforated, and the initial stage of the penetration into a shield was not considered.

In this study, we also use a localized model for impactor–shield interaction and consider all stages of impactor–shield interaction, i.e., not only the stage when an impactor is immersed into a shield but also the initial penetration stage and a stage when an impactor emerges from a shield. The problem is posed as follows: There are two materials with different properties which can be used for manufacturing the plates in a multi-layered shield. The total thicknesses of the plates made of each material are given. The goal is to determine the structure of the target (the order and the thicknesses of the plates from different materials) that provides the maximum ballistic limit velocity of the shield against a normal impact by a

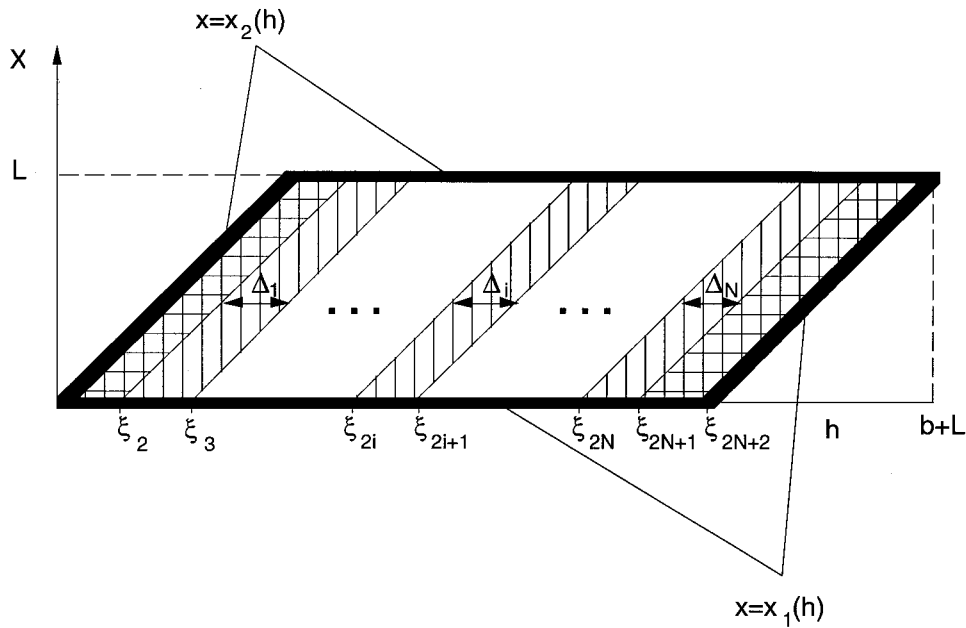


Fig. 2. Scheme of the impactor–shield contact area.

conical impactor. In this study, we not only find the solution to the latter problem without using cumbersome methods of the optimal control theory or nonlinear programming but also rigorously prove that the determined solution is optimum.

In this study, we assume that the impactor–shield interaction at a given location at the surface of the impactor which is in contact with the shield can be described by the following equation:

$$d\vec{F} = [\rho\Omega(u)v^2 + \sigma]\vec{n}^0 dS, u = \cos \beta = -\vec{v}^0 \cdot \vec{n}^0, \tag{1}$$

where $d\vec{F}$ is the force acting at the surface element dS of the impactor along the inner normal vector \vec{n}^0 at a given location at the surface of the impactor, \vec{v}^0 is the unit local velocity vector, β is the angle between the vector \vec{n}^0 and the vector $(-\vec{v}^0)$. Eq. (1) with constant parameters ρ and σ , together with an equation, $\Omega(u)=u^2$, comprise the most widely used phenomenological models for describing the interactions of impactors with homogenous shields manufactured from ductile and some other materials (for details, see e.g. Bunimovich and Dubinsky, 1995; Forrestal et al., 1988, 1990; Nishivaki, 1951; Recht, 1990; Vitman and Stepanov, 1959) where ρ and σ are usually the density and distortion pressure, respectively. In the case of a multi-layered shield, the values of these parameters are determined by the properties of the material of that plate which is in contact with the impactor, and it is assumed that the adjacent layers in the shield do not interact.

Since, according to the employed model, several adjacent plates made from the same material and a single plate with the same total thickness and the same material are equivalent, without the loss of generality we may consider the shields as consisting of alternating plates manufactured from two materials. The notations are shown in Fig. 1. The coordinate h is defined as the distance from the nose of the impactor to the front surface of the target; the coordinate ξ is associated with the target. The plates made from the first material are located between the sections $\xi = \xi_{2i + 1}$ and $\xi = \xi_{2i + 2}$ ($i = 0, 1, \dots, N$), the

plates made from the second material are located between the sections $\xi = \xi_{2i}$ and $\xi = \xi_{2i+1}$ ($i = 1, \dots, N$) where $\xi_1 = 0$, $\xi_{2N+2} = b$, b is the thickness of the shield, the total number of plates is $2N + 1$.

We assume that the shape of the impactor is such that the total force acting on it is directed along the h -axis. Only the nose of the impactor, with length L , interacts with the shield and the impactor may have also a cylindrical part. The part of the lateral surface of the impactor between the cross-sections $x = x_1$ and $x = x_2$ (see Fig. 1) interacts with some plate of the shield where (for details, see Ben-Dor et al., 1997b, 1998b)

$$\bar{x}_1(\bar{h}) = \begin{cases} 0 & \text{if } 0 \leq \bar{h} \leq \bar{b} \\ \bar{h} - \bar{b} & \text{if } \bar{b} \leq \bar{h} \leq \bar{b} + 1 \end{cases}, \quad \bar{x}_2(\bar{h}) = \begin{cases} \bar{h} & \text{if } 0 \leq \bar{h} \leq 1 \\ 1 & \text{if } \bar{b} \leq \bar{h} \leq \bar{b} + 1 \end{cases} \quad (2)$$

and $\bar{x}_1 = \frac{x_1}{L}$, $\bar{x}_2 = \frac{x_2}{L}$, $\bar{h} = \frac{h}{L}$, $\bar{b} = \frac{b}{L}$.

Since the shield consists of different plates, the parameters ρ and σ are functions of the coordinate $\bar{\xi} = \frac{\xi}{L}$,

$$a(\bar{\xi}) = \begin{cases} a_+ & \text{if } \bar{\xi}_{2i+1} \leq \bar{\xi} \leq \bar{\xi}_{2i+2}, \quad i = 0, 1, \dots, N \\ a_\times & \text{if } \bar{\xi}_{2i} \leq \bar{\xi} \leq \bar{\xi}_{2i+1}, \quad i = 1, 2, \dots, N \end{cases} \quad (3)$$

and $a = \rho, \sigma$. Fig. 2 shows (in coordinates h, x) a domain of interaction of an impactor with a shield and sub-domains where an impactor interacts with separate layers in the shield. Hereafter, the parameters associated with the plates manufactured from the first and second materials are denoted with subscripts $+$ and \times , respectively.

The equation of the conical nose part of the impactor can be represented as $r = R_0 \bar{x} \eta(\theta)$, where R_0 is the characteristic size of the base of the impactor, $\bar{x} = \frac{x}{L}$, $\eta(\theta)$ is the function determining the cross-sectional contour of the impactor, $0 < \eta(\theta) \leq 1$, $0 \leq \theta \leq 2\pi$. Since the total force \vec{F} is determined by integrating the local force over the impactor–target contact surface S , the expression for the drag force D can be written as follows:

$$\begin{aligned} D &= \vec{F} \cdot (-v^0) = \iint_S [\rho(\bar{\xi})\Omega(u)v^2 + \sigma(\bar{\xi})]u \, dS \\ &= \tau^2 \int_{x_1(h)}^{x_2(h)} \int_0^{2\pi} [\rho(\bar{h} - \bar{x})\Omega(u)v^2 + \sigma(\bar{h} - \bar{x})]x\eta^2 \, dx \, d\theta, \end{aligned} \quad (4)$$

where

$$u = u(\theta) = \frac{\tau\eta^2}{\sqrt{\eta^2(\tau\eta^2 + 1) + \eta_\theta^2}}, \quad \tau = \frac{R_0}{L}, \quad \eta_\theta = \frac{d\eta}{d\theta}. \quad (5)$$

Using Eq. (4) and dimensionless variables, the equation of motion of the impactor, $M \, d^2h/dt^2 = -D$, can be rewritten as follows:

$$\frac{dv^2}{dh} + f_\rho(\bar{h})v^2 + f_\sigma(\bar{h}) = 0, \quad (6)$$

where the variable v is considered as a function of h , the mass of the impactor M is expressed through its volume V , density γ and a parameter K_M which takes into account the contribution of the cylindrical part of the impactor to its mass and volume (for an impactor without a cylindrical part, $K_M = 1$),

$$f_\sigma = c_\sigma \int_{x_1(\bar{h})}^{x_2(\bar{h})} \bar{x} \sigma(\bar{h} - \bar{x}) d\bar{x}, \quad f_\rho = c_\rho \int_{x_1(\bar{h})}^{x_2(\bar{h})} \bar{x} \rho(\bar{h} - \bar{x}) d\bar{x}, \tag{7}$$

$$c_\sigma = \frac{12}{\gamma K_M}, \quad c_\rho = \frac{c_\sigma}{c}, \quad c = \int_0^{2\pi} \eta^2 d\theta / \int_0^{2\pi} \eta^2 \Omega d\theta. \tag{8}$$

Eq. (6) allows us to determine the ballistic limit velocity, $v_* = \sqrt{w_*}$, which is defined as the initial velocity of the impactor required for its nose to emerge from the target with a zero velocity. Let $v(\bar{h})$ be a linear solution of a differential equation in v^2 Eq. (6) with an initial condition $v(\bar{b} + 1) = 0$. Then $v_* = \sqrt{w_*} = v(0)$ and it can be shown that

$$w_* = \int_0^{\bar{b}+1} f_\sigma(\bar{h}) \exp \left[\int_0^{\bar{h}} f_\rho(\bar{H}) d\bar{H} \right] d\bar{h}. \tag{9}$$

Our goal is to determine $\bar{\xi}_2, \bar{\xi}_3, \dots, \bar{\xi}_{2N+1}$ that provide a maximum w_* under the following conditions:

$$0 = \bar{\xi}_1 \leq \bar{\xi}_2 \leq \bar{\xi}_3 \leq \dots \leq \bar{\xi}_{2N} \leq \bar{\xi}_{2N+1} \leq \bar{\xi}_{2N+2} = \bar{b} = \bar{B}_+ + \bar{B}_\times, \tag{10}$$

$$\sum_{i=0}^N (\bar{\xi}_{2i+2} - \bar{\xi}_{2i+1}) = \bar{B}_+, \quad \sum_{i=1}^N (\bar{\xi}_{2i+1} - \bar{\xi}_{2i}) = \bar{B}_\times, \tag{11}$$

where $\bar{B}_+ = \frac{B_+}{L}$, $\bar{B}_\times = \frac{B_\times}{L}$ and B_+ , B_\times are the given total thicknesses of the plates manufactured of material ‘+’ and ‘×’, respectively.

2. An optimum shield

Let us introduce the function

$$\bar{x}_0(\bar{h}) = \begin{cases} 0 & \text{if } \bar{h} \leq 0 \\ \bar{h} & \text{if } 0 \leq \bar{h} \leq 1 \\ 1 & \text{if } \bar{h} \geq 1 \end{cases} \tag{12}$$

Then, functions f_σ and f_ρ can be represented in the following form:

$$\begin{aligned} f_a(\bar{h}) &= c_a \left[a_+ \sum_{i=0}^N \int_{\bar{x}_0(\bar{h}-\bar{\xi}_{2i+2})}^{\bar{x}_0(\bar{h}-\bar{\xi}_{2i+1})} \bar{x} d\bar{x} + a_\times \sum_{i=1}^N \int_{\bar{x}_0(\bar{h}-\bar{\xi}_{2i+1})}^{\bar{x}_0(\bar{h}-\bar{\xi}_{2i})} \bar{x} d\bar{x} \right] \\ &= c_a \left\{ a_+ \sum_{i=0}^N [\varphi(\bar{h} - \bar{\xi}_{2i+1}) - \varphi(\bar{h} - \bar{\xi}_{2i+2})] + a_\times \sum_{i=1}^N [\varphi(\bar{h} - \bar{\xi}_{2i}) - \varphi(\bar{h} - \bar{\xi}_{2i+1})] \right\} \\ &= c_a [a_+ \psi(\bar{h}) + (a_\times - a_+) \Psi(\bar{h})], \end{aligned} \tag{13}$$

where $a = \rho, \sigma$,

$$\varphi(z) = \int_0^{\bar{x}_0(z)} \bar{x} d\bar{x} = 0.5 [\bar{x}_0(z)]^2, \tag{14}$$

$$\psi(z) = \varphi(z) - \varphi(z - \bar{b}), \Psi(z) = \sum_{i=1}^N \left[\varphi(z - \bar{\xi}_{2i}) - \varphi(z - \bar{\xi}_{2i+1}) \right]. \tag{15}$$

Case. $\rho_+ = \rho_x = \rho$. Using Eqs. (13)–(15), one can rewrite Eq. (9) as follows:

$$w_* = c_\sigma \int_0^{\bar{b}+1} d\bar{h} [\sigma_+ \psi(\bar{h}) + c_\sigma (\sigma_x - \sigma_+) \hat{I}], \tag{16}$$

where

$$\hat{I} = \int_0^{\bar{b}+1} d\bar{h} \Psi(\bar{h}) \exp \left[c_\rho \rho \int_0^{\bar{h}} \psi(\bar{H}) d\bar{H} \right] = \int_0^{\bar{b}+1} d\bar{h} \exp \left[c_\rho \rho \int_0^{\bar{h}} \psi(\bar{H}) d\bar{H} \right] \sum_{i=1}^N \int_{\bar{x}_0(\bar{h}-\bar{\xi}_{2i+1})}^{\bar{x}_0(\bar{h}-\bar{\xi}_{2i})} \bar{x} d\bar{x}. \tag{17}$$

Changing the order of integration after some algebra, we obtain:

$$\begin{aligned} \hat{I} &= \int_0^1 \bar{x} d\bar{x} \sum_{i=1}^N \int_{\bar{x}+\bar{\xi}_{2i}}^{\bar{x}+\bar{\xi}_{2i+1}} d\bar{h} \exp \left[c_\rho \rho \int_0^{\bar{h}} \psi(\bar{H}) d\bar{H} \right] \\ &= \int_0^1 \bar{x} d\bar{x} \sum_{i=1}^N \int_{\bar{x}}^{\bar{x}+\bar{A}_i} d\bar{h} \exp \left[c_\rho \rho \int_0^{\bar{h}+\bar{\xi}_{2i}} \psi(\bar{H}) d\bar{H} \right], \end{aligned} \tag{18}$$

where $\bar{A}_i = \bar{\xi}_{2i+1} - \bar{\xi}_{2i}$ is the thickness of the plate with the number i among the plates manufactured from the second material. Eq. (16) shows that only \hat{I} depends on ξ_j .

Assume that all \bar{A}_i take any values for which the second of Eq. (11) is valid. Then Eq. (18) implies that \hat{I} is an increasing function of $\bar{\xi}_{2i}$ for every $\bar{\xi}_{2i}$. A minimum value of \hat{I} is attained when [see Fig. 3(b)]

$$\bar{\xi}_{2N} = b - \bar{A}_N, \quad \bar{\xi}_{2(N-1)} = b - \bar{A}_N - \bar{A}_{N-1}, \dots, \bar{\xi}_2 = \bar{b} - \sum_{i=N}^1 \bar{A}_i \tag{19}$$

and a maximum value of \hat{I} is attained when [see Fig. 3(a)]

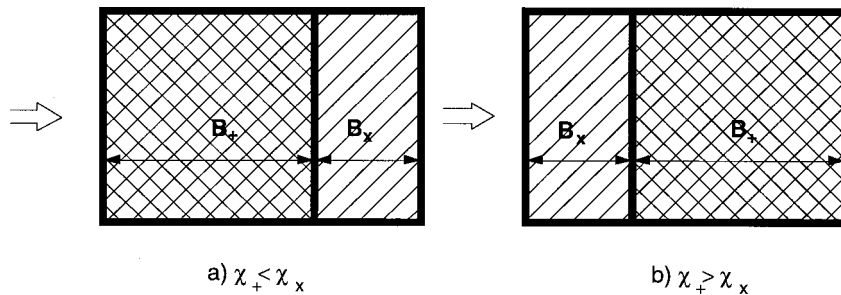


Fig. 3. (a,b). Structure of the optimum shield.

$$\bar{\xi}_2 = 0, \quad \bar{\xi}_4 = \bar{A}_1, \dots, \bar{\xi}_{2N} = \sum_{i=1}^N \bar{A}_i. \tag{20}$$

Thus, a minimum (maximum) value of \hat{I} is obtained when the plates manufactured from the same material are located sequentially and are combined into a single plate, regardless of the initial choice of \bar{A}_i and of the number of layers N . Thus, we found points of extrema of \hat{I} taking into account the variation of all $\bar{\xi}_j$ and N .

Eq. (16) and the results of the above analysis imply that the maximum ballistic limit velocity is obtained when the target consists of just two plates: if $\sigma_+ < \sigma_\times$, then the front plate must be the plate manufactured from material ‘+’ with a thickness B_+ , and the rear plate must be the plate manufactured from material ‘×’ with a thickness B_\times . If $\sigma_+ > \sigma_\times$, then the reverse order of these plates is optimal. Thus, the front plate must always have a lesser value of σ .

There are interesting additional conclusion which can be obtained from the above analysis. Placing any plate with a larger (smaller) value of a parameter σ inside the multi-layered target in the direction of penetration yields an increase (decrease) of the ballistic limit velocity.

Case. $\rho_+ \neq \rho_\times$. Using Eqs. (13)–(15), we can rewrite Eq. (9) in the following form:

$$w_* = c \int_0^{\bar{b}+1} d\bar{h} \left[\frac{\sigma_+}{\rho_+} G(\bar{h}) \frac{dF(\bar{h})}{d\bar{h}} + \frac{\sigma_\times - \sigma_+}{\rho_\times - \rho_+} F(\bar{h}) \frac{dG(\bar{h})}{d\bar{h}} \right], \tag{21}$$

where

$$F(\bar{h}) = \exp \left[c_\rho \rho_+ \int_0^{\bar{h}} \psi(\bar{H}) d\bar{H} \right], \quad G(\bar{h}) = \exp \left[c_\rho (\rho_\times - \rho_+) \int_0^{\bar{h}} \Psi(\bar{H}) d\bar{H} \right]. \tag{22}$$

After integrating the second term in the integral in Eq. (14) by parts, we obtain:

$$w_* = \mu_0 I_0 + \mu_1 I_1, \tag{23}$$

where

$$\mu_0 = c \frac{\sigma_\times - \sigma_+}{\rho_\times - \rho_+}, \quad \mu_1 = c \rho_\times (\chi_\times - \chi_+), \quad \chi_\times = \frac{\sigma_\times}{\rho_\times}, \quad \chi_+ = \frac{\sigma_+}{\rho_+}, \tag{24}$$

$$I_0 = [F(\bar{h})G(\bar{h})]_{\bar{h}=0}^{\bar{h}=\bar{b}+1} = \exp(J_1 + J_2) - 1, \tag{25}$$

$$\frac{J_1}{c_\rho \rho_+} = \int_0^{\bar{b}+1} [\varphi(\bar{H}) - \varphi(\bar{H} - \bar{b})] d\bar{H} = \int_0^{\bar{b}+1} d\bar{H} \int_{\bar{x}_0(\bar{H}-\bar{b})}^{\bar{x}_0(\bar{H})} \bar{X} d\bar{X} = \int_0^1 \bar{X} d\bar{X} \int_{\bar{X}}^{\bar{X}+\bar{b}} d\bar{H} = \frac{\bar{b}}{2}, \tag{26}$$

$$\begin{aligned} \frac{J_2}{c_\rho (\rho_\times - \rho_+)} &= \int_0^{\bar{b}+1} [\varphi(\bar{H} - \bar{\xi}_{2i}) - \varphi(\bar{H} - \bar{\xi}_{2i+1})] d\bar{H} = \int_0^{\bar{b}+1} d\bar{H} \sum_{i=1}^N \int_{\bar{x}_0(\bar{H}-\bar{\xi}_{2i+1})}^{\bar{x}_0(\bar{H}-\bar{\xi}_{2i})} \bar{X} d\bar{X} \\ &= \int_0^1 \bar{X} d\bar{X} \sum_{i=1}^N \int_{\bar{X}+\bar{\xi}_{2i}}^{\bar{X}+\bar{\xi}_{2i+1}} d\bar{H} = \frac{1}{2} \sum_{i=1}^N (\bar{\xi}_{2i+1} - \bar{\xi}_{2i}) = \frac{\bar{B}_\times}{2}, \end{aligned} \tag{27}$$

$$I_1 = \frac{1}{\rho_+ - \rho_\times} \int_0^{\bar{h}+1} d\bar{h} \frac{dF}{d\bar{h}} \exp\left[c_\rho(\rho_\times - \rho_+) \hat{J}(\bar{h})\right], \quad \hat{J}(\bar{h}) = \int_0^{\bar{h}} \Psi(\bar{H}) d\bar{H}. \quad (28)$$

Eqs. (25)–(27) imply that

$$I_0 = \exp\left\{\frac{c_\rho}{2} [\rho_+ \bar{b} + (\rho_\times - \rho_+) \bar{B}_\times]\right\} - 1 = \exp\left[\frac{c_\rho}{2} (\rho_+ \bar{B}_+ + \rho_\times \bar{B}_\times)\right] - 1. \quad (29)$$

Thus, I_0 does not depend on the order and on the thicknesses of the plates in the target.

Let us transform the integral \hat{J} :

$$\begin{aligned} \hat{J} &= \sum_{i=1}^N \left[\int_0^{\bar{h}} \left[\varphi(\bar{H} - \bar{\xi}_{2i}) - \varphi(\bar{H} - \bar{\xi}_{2i+1}) \right] d\bar{H} \right] \\ &= \sum_{i=1}^N \left[\int_{\bar{\xi}_{2i}}^{\bar{h}} \varphi(\bar{H} - \bar{\xi}_{2i}) d\bar{H} - \int_{\bar{\xi}_{2i+1}}^{\bar{h}} \varphi(\bar{H} - \bar{\xi}_{2i+1}) d\bar{H} \right] \\ &= \sum_{i=1}^N \left[\int_0^{\bar{h} - \bar{\xi}_{2i}} \varphi(\bar{H}) d\bar{H} - \int_0^{\bar{h} - \bar{\xi}_{2i+1}} \varphi(\bar{H}) d\bar{H} \right] = \sum_{i=1}^N \int_{\bar{h} - \bar{\xi}_{2i+1}}^{\bar{h} - \bar{\xi}_{2i}} \varphi(\bar{H}) d\bar{H} \\ &= \sum_{i=1}^N \varphi(\bar{H} + \bar{h} - \bar{\xi}_{2i} - \Delta_i) d\bar{H}. \end{aligned} \quad (30)$$

The further analysis is similar for the case $\rho_+ = \rho_\times$. Since φ is a non-decreasing function, \hat{J} is a non-increasing function of $\bar{\xi}_{2i}$ for every $\bar{\xi}_{2i}$. Thus, I_1 is an increasing function of $\bar{\xi}_{2i}$. Using Eq. (23), we conclude that the maximum ballistic limit velocity is obtained when the shield consists of two plates. When $\chi_+ < \chi_\times$, then the front plate must be the plate manufactured from material ‘+’ with a thickness B_+ , and the rear plate must be the plate manufactured from material ‘×’ with a thickness B_\times [see Fig. 3(a)]. When $\chi_+ > \chi_\times$, then the reverse order of these plates is optimal [see Fig. 3(b)]. If $\chi_+ = \chi_\times$, the ballistic limit velocity does not depend on the order of the plates in the target. Relocation of any plate with a larger (smaller) value of the parameter χ inside the shield in the direction of penetration yields an increase (decrease) of the ballistic limit velocity.

Table 1
Experimental results by Radin and Goldsmith (1988)

No. of experiment, i	Structure of a target	$\xi_2^{(i)}$	$\xi_3^{(i)}$	$\xi_4^{(i)}$	$\xi_5^{(i)}$	$\xi_6^{(i)}$	$w_*^{(i)}$	$B_+, B_\times, \Delta_1, \Delta_2$
1	1.6AL–11.7LE–1.6AL	0	1.6	13.3	14.9	14.9	201.3	$B_+ = 11.7, B_\times = 3.2, \Delta_1 = \Delta_2 = 1.6$
2	3.2AL–11.7LE	0	1.6	1.6	3.2	14.9	213	
3	11.7LE–3.2AL	11.7	13.3	14.9	14.9	14.9	190.6	
4	1.6AL–3.2LE–1.6AL	0	1.6	4.8	6.4	6.4	127.5	$B_+ = B_\times = 3.2, \Delta_1 = \Delta_2 = 1.6$
5	3.2AL–3.2LE	0	1.6	1.6	3.2	6.4	125.7	$B_+ = B_\times = 3.2, \Delta_1 = \Delta_2 = 1.6$
6	3.2AL–8.6LE	0	3.2	11.8	—	—	186.7	$B_+ = 8.6, B_\times = 3.2, \Delta_1 = 3.2$
7	8.6LE–3.2AL	8.6	11.8	11.8	—	—	171.4	

Table 2
Theoretical predictions

Compared experiments	Conditions	Prediction
1,2	$\xi_2^{(1)} = \xi_2^{(2)}, \xi_4^{(1)} > \xi_4^{(2)}$	$w_*^{(1)} < w_*^{(2)}$
2,3	$\xi_2^{(2)} < \xi_2^{(3)}, \xi_4^{(2)} < \xi_4^{(3)}$	$w_*^{(2)} > w_*^{(4)}$
4,5	$\xi_2^{(4)} = \xi_2^{(5)}, \xi_4^{(4)} > \xi_4^{(5)}$	$w_*^{(4)} < w_*^{(5)}$
6,7	$\xi_2^{(6)} < \xi_2^{(7)}, \xi_4^{(6)} > \xi_4^{(7)}$	$w_*^{(6)} > w_*^{(7)}$

3. Comparison with the experimental data

Among the published experimental data on ballistic penetration of conical impactors into multi-layered shields consisting of plates manufactured from different materials, we did not find appropriate data sets for comparison with the results obtained above. Therefore, we had to restrict ourselves to the data reported by Radin and Goldsmith (1988) for the shields consisting of plates manufactured from Lexan and Aluminum. The validity of the model Eq. (1) for Lexan was not studied. Therefore, the further analysis serves only to illustrate the obtained results, although they also proved to be valid in this case. Thus, it is conceivable to suggest that Eq. (1) can serve as a good approximation for a wider class of materials, although the coefficients of such approximation cannot always be expressed through the known parameters of the materials.

In a study by Radin and Goldsmith (1988), the ballistic limit velocities for 60-grad conical-nosed projectile penetrating into adjacent layered targets composed of different materials (2024-0 aluminum and Lexan) are investigated. The experiments that are relevant to our study are summarized in Table 1, where the data are presented using the notations of the present study. Subscripts ‘+’ and ‘×’ denote Lexan and aluminum, respectively; all sizes are given in mm, the ballistic limit velocity is given in m/s; the notation 1.6AL–11.7LE–1.6AL denotes 11.7 mm thick Lexan plate sandwiches between two 1.6 mm thick aluminum plates, etc., superscript denotes the number of the experiment. Three series of experiments have been found such that each of the series can be characterized by constants B_+ , B_\times , A_i and N . The results of the analysis are presented in Table 2. If one selects σ as a yield stress parameter ($\sigma_+ = 172$ MPa and $\sigma_\times = 270$ MPa, Radin and Goldsmith, 1988), $\rho_+ = 1290$ kg/m³ and $\rho_\times = 2765$ kg/m³. Then $\chi_+ > \chi_\times$, and the theory predicts that in every series of experiments, the ballistic limit velocity must decrease with an increase of ξ_{2i} . Table 2 shows that this prediction is supported by the experimental results when the difference between the ballistic limit velocities of the compared target is relatively large. The only exception is the case when experiments 4 and 5 are compared, where the difference of the ballistic limit velocities is about 1.5%.

4. Concluding remarks

Ballistic properties of multi-layered targets are studied when the target consists of adjacent non interacting plates manufactured from one of the two possible materials and the total thicknesses of the plates manufactured from every material is fixed. It is found that the displacement of any plate inside the target in the direction of the penetration yields a monotone change in the ballistic limit velocity, and a criterion for the increase or decrease of the ballistic limit velocity which depends on the properties of the materials is found. The performed analysis implies that the maximum ballistic limit velocity is obtained for the two-layered target without alternating the plates manufactured from different materials.

Notably, the obtained results are valid for various conical three-dimensional impactors, e.g. with circular, elliptical or polygonal cross-sections.

Special purpose experiments are required for investigation and validation of the above findings which were obtained using simplified models for the impactor–target interaction.

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